

Introduction

To become accomplished at anything in life takes hard work and effort. Few people are naturally gifted in any area, whether that be in sports, music or academic pursuits. And even for those who are, if they want to be the very best they can be, they need to work hard. The musician puts in hours of practice with scales and finger drills, so that they can play the most difficult pieces of music. The sprinter spends hours in the gym strengthening individual muscles, so that they can run the 100m as fast as possible. The footballer spends hours at training with passing drills, so that they can perform well on the field. In each case, the hard work is done in advance, so that when it matters, the skills and strengths are all there.

This book is a bit like that scales and finger drills book, or the gym exercises, or the passing drills. It has been written to develop skills and strengths in Algebra, so that the learner can put these into practice when it matters, be that in an exam or in more complex mathematical situations.

Often times these skills are not an end in themselves, but rather pave the way for learners to advance in their mathematical competence.

How to use this book

Within each chapter, exercises are differentiated by number. The exercises that begin with a **1** develop the more basic skills. The exercises that begin with a **2** develop skills towards GCSE and National 5 level. The exercises that begin with a **3** are for A-Level and Higher preparation.

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Chapter 1

Simplifying Expressions 1

- Exercise 1.1** Basic addition and subtraction
- Exercise 1.2** Further addition and subtraction
- Exercise 1.3** Multiplication of algebraic terms by numbers
- Exercise 1.4** Multiplication of algebraic terms by letters
- Exercise 1.5** Multiplication of algebraic terms by letters and numbers
- Exercise 1.6** Mixed simplifying 1 – use of BIDMAS
- Exercise 1.7** Division of algebraic terms by letters and numbers
- Exercise 1.8** Mixed simplifying 2 – the four operations
- Exercise 2.1** Further multiplication of terms – The First Law of Indices
- Exercise 2.2** Further division of terms – The Second Law of Indices
- Exercise 2.3** Combination of First and Second Laws of Indices
- Exercise 2.4** Raising powers to powers – The Third Law of Indices
- Exercise 2.5** Negative indices – The Fourth Law of Indices
- Exercise 2.6** Fractional indices
- Exercise 2.7** Mixed simplifying 3 – indices

1.1 Basic Addition and Subtraction

Consider $2x + 3y$, this is called an **algebraic expression**. Within this expression, $2x$ is an **algebraic term**, where 2 is the **coefficient** and x is a **variable**.

When adding and subtracting terms within an algebraic expression, the terms that have the *same variable* go together.

Worked Examples:

1. $a + a + a + a = 4a$

2. $a + a + b + b + b = 2a + 3b$

3. $a + a + b - a = a + b$

Exercise 1.1

Simplify the following:

1. $a + a$

2. $b + b + b$

3. $c + c + c + c + c$

4. $d + d + d + d$

5. $e + e + e + e$

6. $f + f + f + f + f + f$

7. $g + g + g + g + g$

8. $h + h$

9. $i + i + i + i + i + i$

10. $j + j + j$

11. $k + k + k + k + k + k + k$

12. $l + l + l + l + l + l + l + l$

13. $a + a + a + b + b$

14. $c + c + d + d + c$

15. $e + e + f + f + f + f + e$

16. $g + h + g + g + h + h$

17. $j + j + k + k + j$

18. $m + n + m + n + m + m + n$

19. $a + b + a + a + b$

20. $c + c + d + c + d + d$

21. $e + e + e + f + e + f + e + f$

22. $g + h + h + h + g$

23. $j + k + j + k + k$

24. $a + c + b + b + c + a$

25. $a + a + a - a$

26. $b - b + b$

27. $c + c - c + c + c$

28. $d - d + d - d$

29. $e + e + e + e - e - e - e$

30. $f - f + f + f + f + f - f$

31. $a + b + a + a - b$

32. $c + c - d + c - d - d$

33. $e + e + e - f - e + f - e + f$

34. $g + h + h + h - g$

35. $-j - k - j - k + k$

36. $a - c + b - b + c - a$

1.2 Further Addition and Subtraction

Worked Examples:

1. $2a + 3a = 5a$

2. $3a + 5b + 2b = 3a + 7b$

3. $5a + 1 - 4a - 2 = a - 1$

NB: When adding and subtracting algebraic terms that include numbers with no algebraic term (as in example 3), the numbers are treated as normal.

Exercise 1.2

Simplify the following:

1. $3a + a$

2. $2b + 4b$

3. $4c + 3c$

4. $5d + 4d + 3d$

5. $3e + 5e + e$

6. $6f + f + 3f + 2f$

7. $5a + 2b + 3a + b$

8. $6c + 5d + d + 8c$

9. $11e + 12f + f + 10f + 6e$

10. $g + 10h + g + 10g + h$

11. $8j + 17j + 5k + 100k$

12. $14m + n + 5p + 5n + 8p + n$

13. $6a - 3a$

14. $7b - 5b$

15. $8c + 2c - 7c$

16. $12d - 11d + 4d$

17. $14e + 8e - 7e$

18. $7f - 9f$

19. $4a + 3b - 2a - b$

20. $5c - 2d + c - d$

21. $9e + 5f - 10e + f$

22. $8g - 2h - 5g + h$

23. $6j + 2k - 7j - k$

24. $3a - 2b + c + 4a - 6b - 4c$

25. $2x + 4y - 5 - x + 6y$

26. $4c + 2 - 3d + 5 - 2c - d$

27. $5r - 3s + 9 - 3r + 5s - 11$

28. $6p + 9q - 8 + 5p - 12q + 5$

29. $2m - 3n + 9 - 7n + 2m - 2$

30. $4x - 3y - 8x + 5 - 2y - 8$

1.3 Multiplication of Algebraic Terms by Numbers

When multiplying an algebraic term by a number, multiply the **coefficient** of the algebraic term by the number.

NB: If a term has no explicit numerical part (no number in front of the letter), the coefficient is 1. **E.g.** x has a coefficient of 1.

Worked Examples:

1. $a \times 3 = 3a$

2. $2b \times 5 = 10b$

3. $4 \times 7c = 28c$

Exercise 1.3

Simplify the following:

1. $a \times 5$

2. $b \times 6$

3. $c \times 7$

4. $d \times 8$

5. $9 \times e$

6. $13 \times f$

7. $4g \times 2$

8. $7h \times 3$

9. $9i \times 4$

10. $10j \times 2$

11. $4k \times 3$

12. $7 \times 2l$

13. $6m \times 4$

14. $5n \times 3$

15. $6 \times 3p$

16. $4 \times 3q$

17. $8r \times 8$

18. $7 \times 2s$

19. $12 \times 3t$

20. $2u \times 13$

21. $6 \times 7v$

22. $x \times 2 \times 3$

23. $3y \times 4 \times 2$

24. $6 \times 2z \times 5$

Chapter 2

Expanding Brackets

- Exercise 1.1** Expanding single brackets
- Exercise 1.2** Expanding single brackets with negatives
- Exercise 1.3** Addition or subtraction of two or more brackets
- Exercise 2.1** Multiplication of two brackets – both positive
- Exercise 2.2** Multiplication of two brackets – one negative
- Exercise 2.3** Multiplication of two brackets – both negative
- Exercise 2.4** Multiplication of two brackets – perfect square trinomials
- Exercise 2.5** Multiplication of two brackets – mixed exercise
- Exercise 3.1** Multiplication of two brackets – with fractions
- Exercise 3.2** Multiplication of two brackets – with indices
- Exercise 3.3** Multiplication of two brackets – two by three
- Exercise 3.4** Multiplication of three brackets

1.1 Expanding Single Brackets

To expand brackets, the term on the outside of the bracket multiplies each term inside the bracket:

$$a(b + c) = ab + ac$$

Worked Examples:

$$\begin{aligned} 1. \quad & 3(x + 5) \\ & = 3x + 15 \end{aligned}$$

$$\begin{aligned} 2. \quad & 4(x + y) \\ & = 4x + 4y \end{aligned}$$

$$\begin{aligned} 3. \quad & 3x(2x + 7) \\ & = 6x^2 + 21x \end{aligned}$$

$$\begin{aligned} 4. \quad & 2(x + 4) + 2 \\ & = 2x + 8 + 2 \\ & = 2x + 10 \end{aligned}$$

Exercise 1.1

Expand the following, simplify where possible:

1. $a(b + c)$

2. $c(d + e)$

3. $d(e + f)$

4. $e(f + g)$

5. $a(b + 2)$

6. $b(c + 5)$

7. $d(e + 9)$

8. $e(5 + f)$

9. $g(g + h)$

10. $h(k + h)$

11. $2(j + k)$

12. $5(p + q)$

13. $6(m + n)$

14. $7(p + r)$

15. $11(w + s)$

16. $9(e + v)$

17. $4(e + 6)$

18. $8(x + 4)$

19. $3(t + 9)$

20. $12(e + 3)$

21. $7(p + 3)$

22. $10(a + 5)$

23. $4(2w + 6)$

24. $5(2r + 3)$

25. $8(4c + 1)$

26. $9(5h + 4)$

27. $8(3 + 4c)$

28. $5(1 + 9p)$

29. $6(2b + 3e)$

30. $6(6x + 7)$

31. $4a(a + 2)$

32. $3x(x + 6)$

33. $2g(g + 3)$

34. $5t(t + 8)$

35. $8d(d + 5c)$

36. $6y(2y + 3x)$

37. $4(b + 5) + 2$

38. $5(d + 2) + 6$

39. $6(g + 3) + 2g$

40. $7(h + 4) + 13$

41. $2(c + 7) + 5c$

42. $6(k + 5) + 2k$

43. $7(f + 9) + 18$

44. $3(d + 12) + 11$

45. $5a(a + 5) + 2a$

46. $5c(2c + 2) + 4c^2$

47. $3s(s + 2) + 5s^2$

48. $8a(2a + 4) + 6a$

1.2 Expanding Single Brackets with Negatives

To expand brackets, the term on the outside of the bracket multiplies each term inside the bracket:

$$a(b - c) = ab - ac$$

Worked Examples:

$$\begin{aligned} 1. \quad & 3(x - 4) \\ & = 3x - 12 \end{aligned}$$

$$\begin{aligned} 2. \quad & -6(x + y) \\ & = -6x - 6y \end{aligned}$$

$$\begin{aligned} 3. \quad & -3x(5x - 2) \\ & = -15x^2 + 6x \end{aligned}$$

$$\begin{aligned} 4. \quad & 2 - 3(x + 4) \\ & = 2 - 3x - 12 \\ & = -3x - 10 \end{aligned}$$

28. $(k - 5)(5 + k)$

29. $(m + 8)(7 + m)$

30. $(n - 6)(10 - n)$

31. $(2v + w)(v - 2w)$

32. $(w + 2x)(w - 3x)$

33. $(3x + 2y)(4x - 5y)$

34. $(2y + 2)(y - 2z)$

35. $(3x + 2y)(2x + 3z)$

36. $(4x - 3z)(2y + 2x)$

3.1 Multiplication of Two Brackets – With Fractions

Worked Examples:

$$\begin{aligned} 1. \quad & \left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right) \\ &= x^2 + \frac{1}{2}x + \frac{1}{2}x + \frac{1}{4} \\ &= x^2 + x + \frac{1}{4} \end{aligned}$$

$$\begin{aligned} 2. \quad & \left(x + \frac{2}{x}\right)\left(x - \frac{1}{x}\right) \\ &= x^2 - \frac{x}{x} + \frac{2x}{x} - \frac{2}{x^2} \\ &= x^2 - 1 + 2 - \frac{1}{x^2} \\ &= x^2 + 1 - \frac{1}{x^2} \end{aligned}$$

NB: Remember when *multiplying* fractions, multiply the numerators together and denominators together. When *adding* or *subtracting* fractions, a common denominator is needed.

Exercise 3.1

Expand and simplify:

1. $\left(a + \frac{1}{2}\right)(a + 1)$

2. $\left(b - \frac{1}{2}\right)(b + 2)$

3. $\left(c + \frac{1}{2}\right)\left(c - \frac{1}{2}\right)$

4. $\left(d + \frac{1}{3}\right)\left(d + \frac{1}{3}\right)$

5. $\left(e - \frac{2}{3}\right)\left(e - \frac{1}{2}\right)$

6. $\left(f + \frac{1}{4}\right)\left(f - \frac{2}{3}\right)$

7. $\left(g + \frac{3}{4}\right)\left(g + \frac{3}{5}\right)$

8. $\left(h - \frac{1}{5}\right)\left(h - \frac{2}{3}\right)$

9. $\left(j + \frac{1}{4}\right)\left(j - \frac{3}{10}\right)$

10. $\left(2k - \frac{1}{3}\right)\left(k + \frac{1}{8}\right)$

11. $\left(\frac{1}{2}m - \frac{3}{5}\right)\left(3m - \frac{2}{9}\right)$

12. $\left(n - \frac{1}{4}\right)\left(\frac{1}{5}n + \frac{3}{4}\right)$

13. $\left(3p + \frac{3}{4}\right)\left(p + \frac{3}{5}\right)$

14. $\left(2q - \frac{7}{10}\right)\left(q - \frac{1}{4}\right)$

15. $\left(r - \frac{5}{6}\right)\left(2r + \frac{1}{5}\right)$

16. $\left(s - \frac{1}{s}\right)\left(s - \frac{1}{s}\right)$

17. $\left(t - \frac{2}{t}\right)\left(t - \frac{1}{t}\right)$

18. $\left(u - \frac{1}{2u}\right)\left(u + \frac{1}{u}\right)$

19. $\left(v + \frac{3}{v}\right)\left(v + \frac{2}{v}\right)$

20. $\left(\frac{w}{2} - \frac{7}{w}\right)\left(w - \frac{1}{4}\right)$

21. $\left(\frac{2}{x} - x\right)\left(x + \frac{1}{x}\right)$

3.2 Multiplication of Brackets – With Indices

Worked Examples:

$$\begin{aligned} 1. \quad & x^{\frac{1}{2}}\left(x + x^{\frac{1}{2}}\right) \\ &= x^{\frac{3}{2}} + x \\ &= \sqrt{x^3} + x \end{aligned}$$

$$\begin{aligned} 2. \quad & \left(x + x^{\frac{1}{2}}\right)\left(x + x^{\frac{1}{2}}\right) \\ &= x^2 + x^{\frac{3}{2}} + x^{\frac{3}{2}} + x \\ &= x^2 + 2x^{\frac{3}{2}} + x \\ &= x^2 + 2\sqrt{x^3} + x \end{aligned}$$

$$\begin{aligned} 3. \quad & \left(x + x^{\frac{1}{3}}\right)\left(x - x^{-\frac{1}{2}}\right) \\ &= x^2 - x^{\frac{1}{2}} + x^{\frac{4}{3}} - x^{-\frac{1}{6}} \\ &= x^2 - \sqrt{x} + \sqrt[3]{x^4} - \frac{1}{\sqrt[6]{x}} \end{aligned}$$

NB: Remember when *multiplying* expressions with indices, the powers are added together.

e.g. $x^{\frac{1}{2}} \times x^{-\frac{3}{2}} = x^{\frac{1}{2} - \frac{3}{2}} = x^{-1} = \frac{1}{x}$

Exercise 3.2

Expand and simplify:

1. $x^2(x^3 + x)$

2. $y^3(y + 1)$

3. $w^2(w^{-1} + w)$

4. $v(v^2 + v^{-1})$

5. $(z - z^2)(z - z^3)$

6. $(u + u^2)(u - u^{-1})$

7. $a^{\frac{1}{2}}(a^{\frac{1}{2}} + a)$

8. $b^{\frac{1}{2}}(b + b^{-\frac{1}{2}})$

9. $c^{\frac{2}{3}}(c^{\frac{1}{3}} + c)$

10. $d^{\frac{3}{4}}(d^{-\frac{1}{4}} + d)$

11. $(e - e^{\frac{1}{2}})(e - e^{\frac{1}{2}})$

12. $(f + f^{\frac{1}{2}})(f - f^{\frac{1}{2}})$

13. $(g + a^{\frac{1}{2}})(g + a^{-\frac{1}{2}})$

14. $(h - h^{\frac{1}{3}})(h - h^{\frac{1}{3}})$

15. $(j + j^2)(j - j^{-\frac{1}{2}})$

16. $(k^2 - k^{-\frac{1}{2}})(k + k^{-\frac{1}{2}})$

17. $m^{\frac{3}{2}}(m - m^{-\frac{1}{2}})$

18. $n^{\frac{3}{2}}(n - \sqrt[3]{n^2})$

19. $\sqrt[3]{p^4}(p - \sqrt{p})$

20. $\sqrt[3]{q}(q + \frac{1}{\sqrt{q}})$

21. $\sqrt{r}(r^2 + \frac{1}{\sqrt{r}})$

22. $(s^{\frac{1}{2}} - s^{-\frac{1}{2}})^2$

23. $(t^{\frac{1}{3}} - t^{-\frac{1}{3}})^2$

24. $(v^{\frac{2}{3}} - v^{-\frac{2}{3}})^2$

3.3 Multiplication of Brackets – Two by Three

Worked Examples:

1. $(x + 1)(x^2 + 2x + 1)$
 $= x^3 + 2x^2 + x + x^2 + 2x + 1$
 $= x^3 + 3x^2 + 3x + 1$

2. $(3x - 2)(x^2 + 2x - 5)$
 $= 3x^3 + 6x^2 - 15x - 2x^2 - 4x + 10$
 $= 3x^3 + 4x^2 - 19x + 10$

NB: When multiplying brackets together, each term in the first bracket must be multiplied by each term in the second bracket.

Exercise 3.3

Expand and simplify:

1. $(a + 1)(a^2 + 2a + 1)$

2. $(b + 2)(b^2 + b + 1)$

3. $(c + 2)(c^2 + 3c + 1)$

4. $(d + 1)(d^2 + 5d + 3)$

5. $(e - 4)(e^2 + 2e + 3)$

6. $(f - 5)(f^2 + 3f - 1)$

7. $(g - 2)(g^2 - 3g + 1)$

8. $(h + 1)(h^2 - 5h - 1)$

9. $(j + 3)(j^2 - 4j + 5)$

10. $(k - 4)(k^2 + 4k - 2)$

11. $(m - 5)(m^2 + 5m + 1)$

12. $(n - 3)(n^2 - n - 4)$

13. $(p + 3)(p^2 + 5p - 1)$

14. $(q + 3)(q^2 - q + 6)$

15. $(r - 3)(r^2 + 4r - 3)$

16. $(s + 3)(2s^2 - 4s + 1)$

17. $(3t + 8)(2t^2 + 5t + 1)$

18. $(3u - 2)(4u^2 + 2u - 1)$

19. $(2v - 2)(v^2 - 5v - 1)$

20. $(w - 3)(4w^2 - 2w + 1)$

21. $(3x + 5)(5x^2 + 3x - 7)$

22. $(2y - 7)(5y^2 + 2y - 9)$

23. $(3z + 8)(5z^2 - 1)$

24. $(5z - 3)(3z^2 - 5)$

3.4 Multiplication of Three Brackets

Worked Examples:

$$\begin{aligned}1. & (x+3)(x-2)(x+4) \\ &= (x+3)(x^2+2x-8) \\ &= x^3+2x^2-8x+3x^2+6x-24 \\ &= x^3+5x^2-2x-24\end{aligned}$$

$$\begin{aligned}2. & (x+3)^3 \\ &= (x+3)(x+3)(x+3) \\ &= (x+3)(x^2+6x+9) \\ &= x^3+6x^2+9x+3x^2+18x+27 \\ &= x^3+9x^2+27x+27\end{aligned}$$

NB: By expanding the second two brackets, the expression can then be expanded as in Exercise 3.3 (above).

Exercise 3.4

Expand and simplify:

1. $(a+2)(a+3)(a+5)$
2. $(b+2)(b+1)(b+2)$
3. $(c+4)(c+1)(c+6)$
4. $(d+1)(d-3)(d-5)$
5. $(e-4)(e+3)(e-5)$
6. $(f-5)(f-1)(f-3)$
7. $(g-2)(g+5)(g-4)$
8. $(h+1)(h-1)(h+1)$
9. $(j-3)(j+5)(j-7)$
10. $(k-4)(k-2)(k-7)$
11. $(m-5)(m+1)(m-9)$
12. $(n-3)(n-4)(n+9)$
13. $(p-2)^3$
14. $(q+3)(q+6)(q-3)$
15. $(r-5)^3$
16. $(s+4)^3$
17. $(t-6)^3$
18. $(u-2)(2u-1)(u+3)$
19. $(2v-2)(v+1)^2$
20. $(w-3)^2(w+1)$
21. $(2x+5)(x-7)(2x-1)$
22. $(2y-5)(2y-3)^2$
23. $(3z+4)^3$
24. $(z-3)(z^2-5)^2$