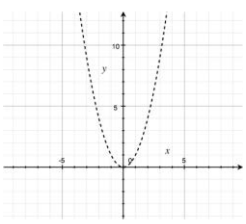
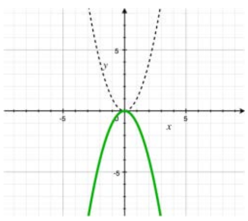
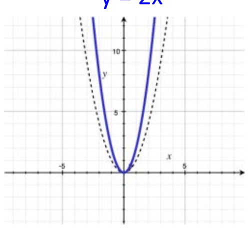
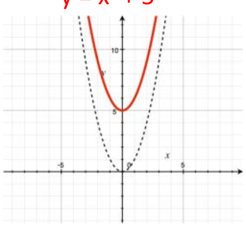
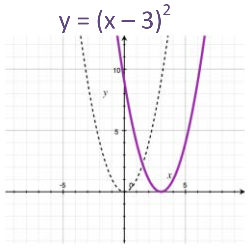
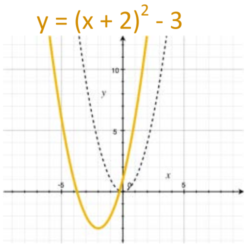
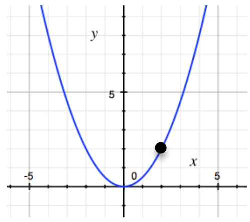
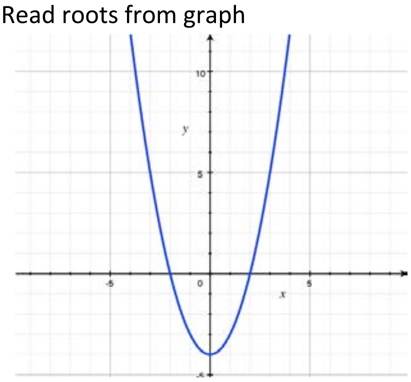
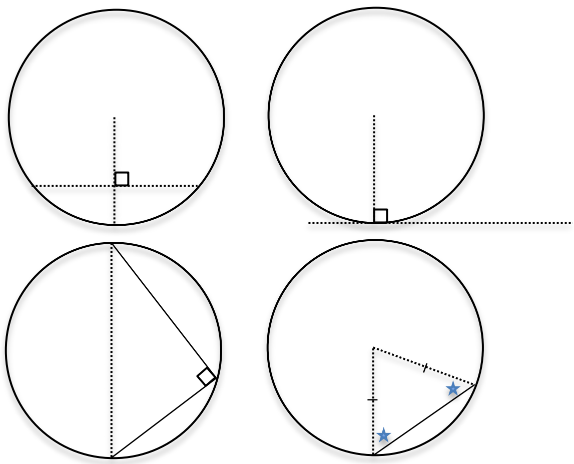
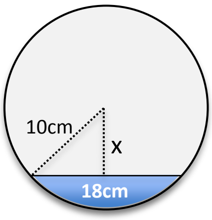
National 5 Learning Checklist - Relationships

Topic	Skills	Extra Study / Notes			
Straight Line					
Gradient	• Represented by ***m*** • Measure of steepness of slope • Positive gradient – the line is increasing • Negative gradient – the line is decreasing				
Y-intercept	• Represented by ***c*** • Shows where the line cuts the y-axis • Find by making x = 0				
Find the gradient of a line joining two points	Know that gradient is represented by the letter ***m*** **Step 1:** Select two coordinates **Step 2:** Label them (x_1, y_1) (x_2, y_2) **Step 3:** Substitute them into gradient formula **e.g.** (-4, 4), (12, -28) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-28) - 4}{12 - (-4)} = \frac{-32}{16} = -2$				
Find equation of a line (from gradient and y-intercept)	**Step 1:** Find gradient ***m*** **Step 2:** Find y-intercept ***c*** **Step 3:** Substitute into y = *m*x + ***c*** (see above for definitions)				
Find equation of a line (from two points)	Use this when there are only two points (i.e. no y-intercept) **Step 1:** Find gradient **Step 2:** Substitue into y – **b** = *m*(x – **a**) where (**a**, **b**) are taken from either one of the points				
Rearrange equation to find gradient and y-intercept	**e.g.** 3y + 6x = 12 3y = -6x + 12 y = -2x + 4 *m* = -2, ***c*** = 4				
Sketch lines from their equations	**Step 1:** Rearrange equation to the form y = *m*x + ***c*** (see note above) **Step 2:** Draw a table of points **Step 3:** Plot points on coordinate axes				
Solving Equations / Inequations					
Solving Equations	Use suitable method: **e.g.** 5(x + 4) = 2(x – 5) 5x + 20 = 2x – 10 5x = 2x – 30 3x = 30 x = 10				
Solving inequations	Solve the same way as equations. **NB:** When dividing by a negative change the sign: **e.g.** -3x ≤ 15 x ≥ -5				
Simultaneous Equations					
Solve by sketching lines	**Step 1:** Rearrange lines to form y = *m*x + ***c*** **Step 2:** Sketch lines using table of points (as above) **Step 3:** Find coordinate of point of intersection				
Solve by substitution	This works when one or both equations are of the form y = ax + b **e.g.** Solve 3x + 2y = 17 ——① y = **x + 1** ——② Sub equation 2 into 1: 3x + 2(**x + 1**) = 17 5x + 2 = 17 x = 3 so y = 3 + 1 = 4				

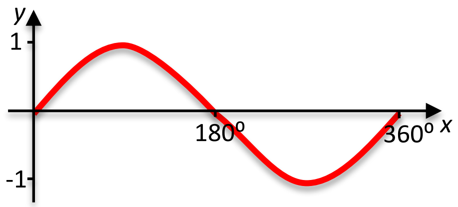
Simultaneous Equations Contd.					
Solve by Elimination	**Step 1:** Scale equations to make one unknown equal with opposite sign. **Step 2:** Add Equations to eliminate equal term and solve. **Step 3:** Substitute number to find second term. **e.g.** $4a + 3b = 7$ ① $2a - 2b = -14$ ② ①x 2 $8a + 6b = 14$ ③ ②x 3 $6a - 6b = -42$ ④ ③+④ $14a = -28$ $a = -2$ substitute $a = -2$ into ① $4(-2) + 3b = 7$ $3b = 15$ $b = 5$ Ans. $a = -2, b = 5$				
Form Equations	Form equations from a variety of contexts to solve for unknowns				
Change the Subject					
Linear Equations	Rearrange equations change the subject: **e.g.** $D = 4C - 3$ [**C**] $D + 3 = 4C$ $\frac{D+3}{4} = C$ $C = \frac{D+3}{4}$ $y = 5(z + 6)$ [**z**] $\frac{y}{5} = z + 6$ $\frac{y}{5} - 6 = z$ $z = \frac{y}{5} - 6$				
Equations with powers or roots	**e.g.** $V = \pi r^2 h$ [**r**] $\frac{V}{\pi h} = r^2$ $r = \sqrt{\frac{V}{\pi h}}$				
Quadratic Functions					
Quadratics and their equations	$y = x^2$  $y = -x^2$  $y = 2x^2$  $y = x^2 + 5$  $y = (x - 3)^2$  $y = (x + 2)^2 - 3$ 				

Equations of quadratics $y = kx^2$	**Step 1:** Identify coordinate from graph **Step 2:** Substitute into $y = kx^2$ **Step 3:** Solve to find k **e.g.** **Coordinate:** (2, 2) **Substitution:** $2 = k(2)^2$ $2 = 4k$ $k = 0.5$ **Quadratic:** $y = 0.5x^2$  				
Sketching Quadratics $y = k(x + a)^2 + b$	**Step 1:** Identify shape, if k = 1 then graph is +ve or if k = -1 then the graph is -ve **Step 2:** Identify turning point (-a, b) **Step 3:** Sketch axis of symmetry x = -a **Step 5:** Find y-intercept (make x = 0) **Step 4:** Sketch information				
Sketching Quadratics (Harder) $y = (x + a)(x - b)$	**Step 1:** Identify shape (+ve or -ve) **Step 2:** Identify roots (x-intercepts) x = -a, x = b **Step 3:** Find y-intercept (make x = 0) **Step 4:** Identify turning point **e.g.** $y = (x + 4)(x - 2)$ +ve graph ∴ Minimum turning point Roots: x = 2, x = -4 y-intercept: $y = (0 + 4)(0 - 2) = -8$ Turning Point (-1, -9) (see below) **NB:** Turning point is halfway between roots. x-coord = $(2 + (-4)) \div 2 = -1$ y-coord = $(-1 + 4)(-1 - 2) = -9$				
Solving Quadratics (finding roots) – Algebraically	**Step 1:** Factorise quadratic **Step 2:** Set each factor equal to zero **Step 3:** Solve each factor to find roots **e.g.** $y = x^2 + 4x$ $x(x + 4) = 0$ $x = 0$ or $x + 4 = 0$ $x = 0$ or $x = -4$ $y = x^2 - 5x - 6$ $(x - 6)(x + 1) = 0$ $x - 6 = 0$ or $x + 1 = 0$ $x = 6, x = -1$				
Solving Quadratics (finding roots) – Graphically	Read roots from graph  x = 2, x = -2				
Solving Quadratics – Quadratic Formula	When asked to solve a quadratic to a number of decimal places use the formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $y = ax^2 + bx + c$				

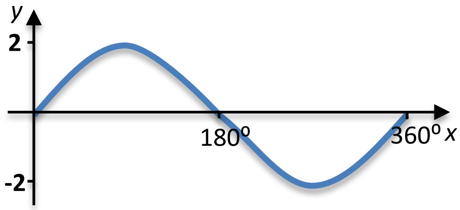
	e.g. Solve $y = x^2 - 6x + 2$ to 1 d.p. **a** = 1 **b** = -6 **c** = 2 $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4\times1\times2}}{2\times1}$ $x = \frac{6 \pm \sqrt{28}}{2}$ $x = \frac{6 + \sqrt{28}}{2}$ $x = \frac{6 - \sqrt{28}}{2}$ $x = 5.6$ $x = 0.4$				
Discriminant	**$b^2 - 4ac$** where $y = ax^2 + bx + c$ The discriminant describes the nature of the roots $b^2 - 4ac > 0$ two real roots $b^2 - 4ac = 0$ equal roots (tangent to axis) $b^2 - 4ac < 0$				
Using the Discriminant	**Example 1:** Determine the nature of the roots of the quadratic $y = x^2 + 5x + 4$ **Solution:** **a** = 1 , **b** = 5, **c** = 4 $b^2 - 4ac = 5^2 - 4 \times 1 \times 4 = 25 - 16 = 9$ Since $b^2 - 4ac > 0$ the quadratic has two real roots. **Example 2:** Determine p, where $x^2 + 8x + p$ has equal roots **Solution:** $b^2 - 4ac = 0$ $8^2 - 4 \times 1 \times p = 0$ $64 - 4p = 0$ $64 = 4p$ $P = 16$				
Properties of Shapes					
Circles					
Pythagoras	Use Pythagoras Theorem to solve problems involving circles and 3D shapes. **e.g.** Find the depth of water in a pipe of radius 10cm.  *r* is the radius $x^2 = 10^2 - 9^2$ $x^2 = \ldots$ $x = 4.4$cm Depth = 10 – 4.4 = 5.6cm				
Similar Shapes					
Linear Scale Factor	$Linear.Scale.Factor = \frac{New.Length}{Original.Length}$				

Area Scale Factor	$Area.Scale.Factor = \left(\frac{New.Length}{Original.Length}\right)^2$				
Volume Scale Factor	$Volume.Scale.Factor = \left(\frac{New.Length}{Original.Length}\right)^3$				
Trigonometry					
Trig Graphs – Sine Curve	**y = *a*sin *bx* + *c*** ***a*** = maxima and minima of graph ***b*** = no. of waves between 0 and 360° ***c*** = movement of graph vertically				

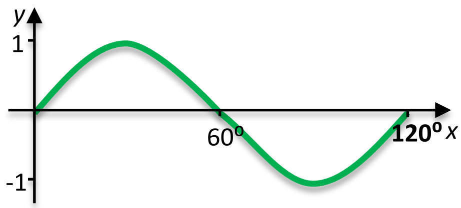
y = sin *x* maxima and minima 1 and -1, period = 360°



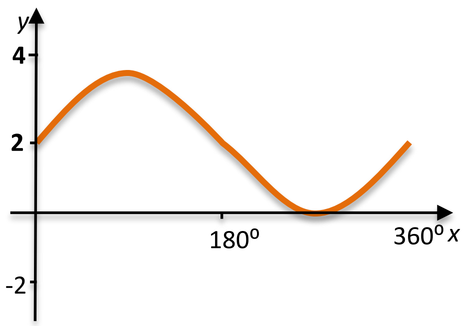
y = 2sin x



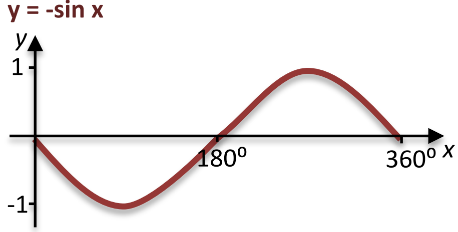
y = sin 3x



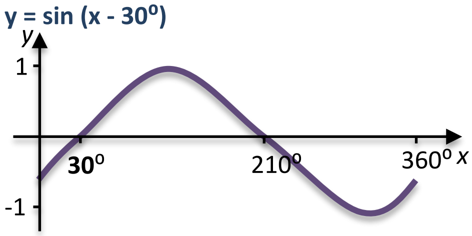
y = 2sin x + 2

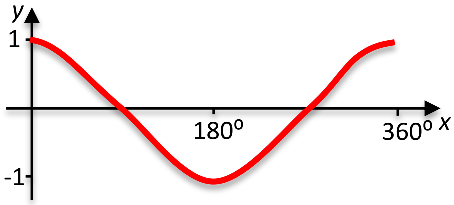
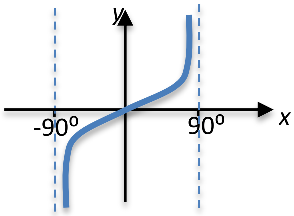
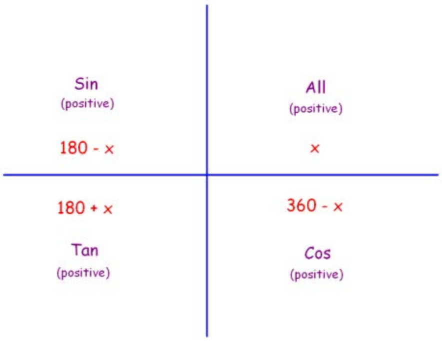


y = -sin x



y = sin (x - 30°)



Trig Graphs – Cosine Curve	**$y = a\cos bx + c$** ***a*** = maxima and minima of graph ***b*** = no. of waves between 0 and 360° ***c*** = movement of graph vertically				
	y = cos x maxima and minima 1 and -1, period = 360°  The same transformations apply for Cosine as Sine (above)				
Trig Graphs – Tan Curve	**y = tan x** no maxima or minima, period = 180°  				
Solving Trig Equations	Know the CAST diagram  **Memory Aid: A**ll **S**tudents **T**ake **C**are				
	Use the diagram above to solve trig equations: **Example 1:** Solve $2\sin x - 1 = 0$ $2\sin x = 1$ $\sin x = ½$ $x = \sin^{-1}(½)$ $x = 30^\circ, 180^\circ - 30^\circ$ $x = 30^\circ, 150^\circ$ **Example 2:** Solve $4\tan x + 5 = 0$ $4\tan x = -5$ $\tan x = -5/4$ **NB:** tan x is negative so there will be solutions in the second and fourth quadrant $x = = \tan^{-1}(5/4)$ $x_{acute} = 51.3^\circ$ $x = 180^\circ - 51.3^\circ, 360 - 51.3^\circ$ $x = 128.7^\circ, 308.7^\circ$				
Trig Identities	Know: $\sin^2 x + \cos^2 x = 1$ $\therefore \sin^2 x = 1 - \cos^2 x$ *and* $\cos^2 x = 1 - \sin^2 x$ and $\tan x = \dfrac{\sin x}{\cos x}$				
	Use the above facts to show one trig function can be another. Start with the left hand side of the identity and work through until it is equal to the right hand side.				